2

TRANSMISSION LINES

2.1 INTRODUCTION

THE problem of the distortionless transmission of signals over wires is a familiar one in electrical communication systems. In the nanosecond range the problem becomes acute, owing to the large bandwidths involved, and is met with even when the distances encountered are as small as a few inches. Difficulties arise in the layout and construction of electronic units due to the series self inductance and shunt capacitance to earth of every piece of connecting wire employed.

The theory of transmission lines, which has been well developed in the field of radio, indicates how such problems may be solved. We shall find that a pulse can be propagated down a line without distortion and that the input impedance of any length of a properly terminated line is a pure resistance. Upon these two properties hinge most applications of lines and we can see that all inter-connexions must either be extremely short in length or must form a properly designed transmission line, which may then be of any length, working between the correct terminal impedances.

A survey of conventional transmission line theory, using the method of Laplace transforms, will be given here since lines find wide application in the equipments with which we are concerned and also form the basis of several devices which are peculiarly suited for use in the nanosecond range.

2.2 UNIFORM RECTILINEAR LINES

2.2.1 Summary of Properties.

Treatments based on MAXWELL's equations are given in the literature (see for example RAMO and WHINNERY [2001], SCHELKUNOFF [2002]) and some universal properties of the electric and

magnetic wave fields and derived in Appendix I. The following is a summary of general results which (except (v)) apply to lines consisting of sensibly perfect conductors:

(i) A transmission line must consist of at least two* separate conductors. If these are parallel to one another and of constant, though arbitrary, cross-section along the line the system may be referred to as a rectilinear uniform line.

(ii) The solution of MAXWELL's equations, with the appropriate boundary conditions, corresponding to the simplest possible

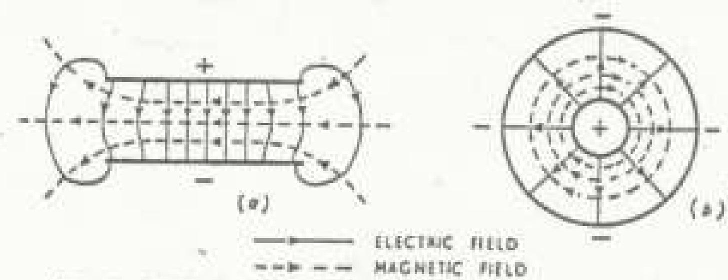


Fig. 2.1. Field distributions in transverse plane for (a) parallel strip and (b) coaxial transmission lines. (Wave travelling into plane of diagram.)

field pattern, shows the existence of a travelling wave (in either direction) in the principal or transverse electromagnetic mode. Properties possessed by waves in this mode are:

(iii) The electric and magnetic fields are always mutually perpendicular and at right-angles to the direction of propagation.

(iv) The characteristic impedance (see later) is a pure resistance and its value is independent of frequency.

(v) When losses are present (see § 2.2.6) the attenuation always increases with increasing frequency.

(vi) The electric and magnetic field distributions, in any transverse plane, are the same as those for electrostatic charges situated on the conductors and for direct currents† flowing through them respectively. Figs. 2.1(a) and (b) show the

* For information on multiple conductor lines see articles by BOAST [2003], BRAZMA [2004, 2005], BRUDERLINK [2006], FRANKEL [2007], and KLUSS [2008].

† Because of the skin effect the high frequency currents flow only in the surface layers of the conductors; the analogous direct currents must therefore be supposed to be confined to such layers, which, for perfect conductors, are infinitely thin.

respective configurations in the case of a parallel strip transmission line and a circular coaxial line.

(vii) It is possible to think in terms of a potential difference across the two conductors and equal and opposite currents flowing along them. A shunt capacitance C per unit length and a series inductance L per unit length can be introduced the values of which may be determined by static or quasi-stationary theory (see previous footnote).

(viii) The velocity of propagation is independent of frequency; in particular, there does not exist any low frequency cut-off as obtains in a waveguide. The speed is equal to that of a plane wave in an infinite volume of the dielectric which fills the space between the conductors. If T is the time delay per unit length then

$$T = \sqrt{\varepsilon\mu} = \frac{10}{3}\sqrt{\kappa_e \kappa_m} \text{ nsec/m} \tag{2.1}$$

where $\kappa_e = \varepsilon/\varepsilon_0$ is the relative permittivity of the medium, and $\kappa_m = \mu/\mu_0$ is the relative permeability (the quantities ε_0 and μ_0 refer to free space).

(ix) We shall see that the characteristic impedance Z_0 is given by $\sqrt{L/C}$ and also that $T = \sqrt{LC}$ thus, by relation 2.1, if either L or C is known Z_0 may be determined. The calculation of characteristic impedances accordingly reduces to the determination of either L or C in the static case.

The application of distributed circuit theory is justified over regions where the line is uniform, or when the transverse dimensions

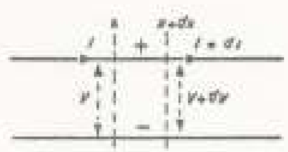


Fig. 2.2. Transmission line analysis.

change only slowly with distance along the line. The effect of discontinuities, however, must be investigated by means of electromagnetic theory. We shall now proceed to elaborate some of the above statements.

2.2.2 Analysis.

Turning to the conventional distributed circuit analysis of a transmission line (fig. 2.2), let $V_x(t)$ and $I_x(t)$ be the voltage and current at time t at a point x on the line. By considering the potential fall across a small distance dx due to the current flowing in the series inductance $L \,.\, dx$ we have

$$\frac{d\bar{V}_x}{dx} = -pL \,.\, \bar{I}_x \tag{2.2}$$

where the Laplace transforms,* with respect to time, of the voltage and current have been taken. Consideration of the drop in current over the element dx due to the voltage acting across the shunt capacitance $C \,.\, dx$ shows that

$$\frac{d\bar{I}_x}{dx} = -pC \,.\, \bar{V}_x \tag{2.3}$$

On differentiating both sides of equation 2.2 with respect to x and substituting for $d\bar{I}_x/dx$ from relation 2.3 we obtain

$$\frac{d^2\bar{V}_x}{dx^2} = p^2LC \,.\, \bar{V}_x \tag{2.4}$$

provided L is independent of x.

Similarly it follows that

$$\frac{d^2\bar{I}_x}{dx^2} = p^2LC \,.\, \bar{I}_x \tag{2.5}$$

provided C is independent of x.

It is easily verified that

$$\bar{V}_x = \vec{\bar{V}}_0 e^{-pTx} = \vec{\bar{V}}_x \quad \text{say} \tag{2.6}$$

and

$$\bar{V}_x = \overleftarrow{\bar{V}}_0 e^{pTx} = \overleftarrow{\bar{V}}_x \quad \text{say} \tag{2.7}$$

are each solutions of equation 2.4 where $\vec{\bar{V}}_0$ and $\overleftarrow{\bar{V}}_0$ are constants, provided both L and C are independent of x (the significance of the arrows will appear in a moment). We have put

$$T = \sqrt{LC} \tag{2.8}$$

* A Laplace transform treatment of transmission lines has been given by Waidelich [2009].

The general solution of equation 2.4 is the sum of the solutions 2.6 and 2.7 and accordingly

$$\bar{V}_x = \vec{\bar{V}}_x + \overleftarrow{\bar{V}}_x = \vec{\bar{V}}_0 e^{-pTx} + \overleftarrow{\bar{V}}_0 e^{pTx} \qquad (2.9)$$

On taking the inverse Laplace transform (see Table 1.1, page 10) this becomes

$$V_x(t) = \vec{V}_x(t) + \overleftarrow{V}_x(t) = \vec{V}_0(t - Tx) + \overleftarrow{V}_0(t + Tx) \qquad (2.10)$$

provided $t - Tx > 0$.

The first term on the right hand side of the solution 2.10 shows that the voltage at the point x at time $t + Tx$ is the same as that at $x = 0$ at time t i.e. we have propagation, without distortion, with delay time T per unit length. The proviso $t - Tx > 0$ indicates that we must not use this term to give the effect at x until the wave has had time to travel from $x = 0$. Similarly the second term represents a wave travelling with the same speed but along the negative direction of x.

Exactly the same arguments apply for the solution of the current wave equation 2.5. We have only to introduce two new constants and in place of equations 2.6, 2.7 and 2.9 we have

$$\bar{I}_x = \vec{\bar{I}}_x + \overleftarrow{\bar{I}}_x \qquad (2.11)$$

where

$$\vec{\bar{I}}_x = \vec{\bar{I}}_0 e^{-pTx} \qquad (2.12)$$

and

$$\overleftarrow{\bar{I}}_x = \overleftarrow{\bar{I}}_0 e^{pTx} \qquad (2.13)$$

The complete solutions 2.9 and 2.11 must satisfy equation 2.2 (or 2.3) for all values of x. If the quantity Z_0 is defined by

$$Z_0 = \sqrt{L/C} \qquad (2.14)$$

it is easily shown that

$$\frac{\vec{\bar{V}}_0}{\vec{\bar{I}}_0} = \frac{\vec{\bar{V}}_x}{\vec{\bar{I}}_x} = -\frac{\overleftarrow{\bar{V}}_0}{\overleftarrow{\bar{I}}_0} = -\frac{\overleftarrow{\bar{V}}_x}{\overleftarrow{\bar{I}}_x} = Z_0 \qquad (2.15)$$

Since Z_0 does not involve p the operation of taking the inverse transform throughout relation 2.15 consists simply of the removal of the bars. We therefore see that the characteristic impedance Z_0 (which is purely resistive and independent of frequency) gives the ratio of the voltage to the current in the forward going wave component; the ratio is the same for the backward travelling wave but with the sign changed.

2.2.3 General.

Fig. 2.1 shows only two of many possible forms of transmission line; papers by Anderson [2010], Barclay and Spangenberg [2011], Brown [2012], Buchholz [2013], Craggs and Tranter [2014], Emmrich [2016], Frankel [2017, 2018], Gans [2019], Gent [2020], Jones [2021], Landsberg [2022], Meinke [2023], Omar and Miller [2024], Parzen [2025], Quilico [2026], Re Qua [2027], Rothe [2028], Shebes [2029], Snow [2030], Tomcio [2031], Tseitlin [2032, 2033, 2034], Wheeler [2035, 2036], Wise [2037] and Wong [2038], contain information on lines having various cross-sectional shapes (including curvilinear lines). The characteristic impedances of a number of configurations are given in Appendix III.

In the majority of applications we are concerned with an unbalanced system, that is, one in which voltages with respect to earth are relevant. When such is the case a single insulated conductor surrounded completely by a metal shield may be employed. The shield acts both as the second conducting element and as a screen. Owing to the skin effect, the current in the shield flows entirely on its inner surface and the outside of the shield is completely "dead" and may be earthed anywhere or everywhere without affecting operation (but see § 3.5). For details of the design, construction and properties of coaxial cables the reader may consult papers by Kenney [2039], Smith [2040], Stanford and Quarmby [2041], Zimmerman [2042] and the Conference Report [2043].

When the system is balanced, two insulated conductors may be employed, again completely surrounded by a shield.

Totally screened lines are usually preferable to open types since the latter are subject to a frequency dependent loss of energy by radiation and unwanted signals may be picked up in nearby circuits (or vice-versa). When the latter objections are irrelevant, however, the so-called strip line techniques may be employed with advantage. The unbalanced version of this type of line consists of a metallic strip in proximity to a metallic earth plane, and, in many cases, separated from it by a slab of dielectric material. The characteristic impedance depends upon the width of the strip and the separation from the ground plane (see Appendix III) and a wide range (say

30–300 Ω) may be covered quite conveniently. The line has the advantage of accessibility for the connexion of tapping points or shorting bridges, which may be continuously variable in position, and also lends itself to a printed circuit type of construction. Further information may be sought in papers by BATES [2044], BOWNESS [2045], COHN [2046], CHISHOLM [2047], DAHLMAN [2048], DUKES [2049, 2050], HAYT [2051] and PARK [2052]. Design details for coaxial to strip line transitions are given by SION [2053].

2.2.4 Terminations and Discontinuities.

We have been concerned in the previous sections with the properties of the main body of the line and now investigate the effect of discontinuities which may be present.

2.2.4.1 Arbitrary Termination

The most important, and most obvious, discontinuities occur at the two ends of a finite length of line. Let us suppose that an arbitrary impedance Z, a function of p, is connected across the right hand, or far, end of the line i.e. the end remote from the generator feeding the line. If the amplitudes of the waves, just at the termination, are as indicated in fig. 2.3(a) OHM's law gives

$$Z = \frac{\vec{V} + \overleftarrow{V}}{\vec{I} + \overleftarrow{I}} \tag{2.16}$$

Substituting for the currents in terms of the voltages by means of relations 2.15 we find for the voltage reflection coefficient

$$\rho = \frac{\overleftarrow{V}}{\vec{V}} = \frac{Z - Z_0}{Z + Z_0} \tag{2.17}$$

Three particular cases are of special interest:

(a) If $Z = Z_0$ then $\rho = 0$. There is no reflected wave and all the power is absorbed in the pure resistance Z. The line is said to be matched or properly terminated.

(b) If $Z = 0$, corresponding to a short circuit, then $\rho = -1$ and the reflected voltage wave is of the same amplitude as the incident one but opposite in sign.

(c) If $Z \to \infty$, as is the case when the line is open-circuited, $\rho = 1$ and reflection occurs without change in phase.

We may at once apply relation 2.17 to the case of the junction between two lines of different characteristic impedances (fig. 2.3(b)). No loss in generality is incurred if we suppose that there is only one incident wave i.e. that travelling to the right in line 1. Relations 2.15 show that the input impedance $\vec{V}_2/\vec{I}_2$ of the line 2 is simply Z_{02}

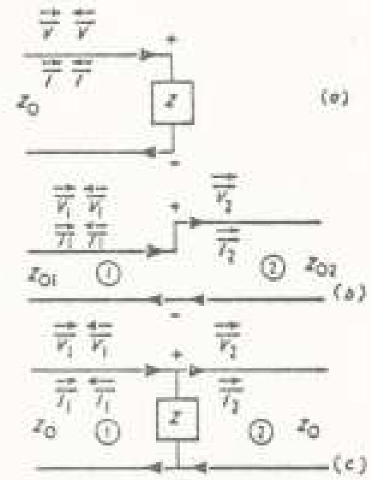


Fig. 2.3. Major discontinuities in a transmission line.
(a) Arbitrary termination.
(b) Junction of two lines of different characteristic impedances.
(c) Shunt impedance connected across a uniform line.

and this quantity is accordingly substituted for Z in the previous equation yielding

$$\rho = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}} \tag{2.18}$$

In the absence of any extra lumped series impedance at the junction we must have

$$\vec{V}_1 + \overleftarrow{V}_1 = \vec{V}_2 \tag{2.19}$$

and accordingly a voltage transmission coefficient can be defined as

$$\frac{\vec{V}_2}{\vec{V}_1} = 1 + \frac{\overleftarrow{V}_1}{\vec{V}_1} = 1 + \rho \tag{2.20}$$

When $Z_{01} = Z_{02}$ then $\rho = 0$ and the transmission coefficient equals unity; no reflection occurs at the junction of two lines of the same impedance and the transmission is complete.

Let us now consider the case of an arbitrary impedance Z connected across a uniform line at some intermediate point (fig. 2.3(c)). The effective impedance terminating the left hand section of line is equal to $ZZ_0/(Z + Z_0)$ and equation 2.17 gives

$$\rho = \frac{-Z_0}{2Z + Z_0} \tag{2.21}$$

In this case the voltage is continuous over the discontinuity and relations 2.19 and 2.20 are again satisfied.

Similar arguments may be applied in the determination of current reflection and transmission coefficients for the cases indicated in fig. 2.3. In case (c) of this figure the current is not continuous, however, over the discontinuity and the equation in current analogous to 2.19 is not true (similarly relation 2.19 for the voltage would not obtain if a lumped series impedance were inserted in the line). The coefficients are found in all cases by employing relations 2.15 in each section of line on either side of the discontinuity and applying OHM's law and the appropriate continuity relation.

The presence of a discontinuity on the line with its attendant reflection represents an effective attenuation of the main wave. If the discontinuity is brought about by purely resistive impedances, as in fig. 2.3(b) or in fig. 2.3(c) when Z is a resistance, the reflection coefficient is independent of p and of frequency; no distortion occurs. When the discontinuity involves reactances ρ is a function of p and the "attenuation" is accompanied by distortion.

In the realm of sinusoidal oscillations reflection coefficients are usually measured in terms of the standing wave ratio S. The following relation obtains

$$S = \frac{|\vec{\mathcal{V}}| + |\overleftarrow{\mathcal{V}}|}{|\vec{\mathcal{V}}| - |\overleftarrow{\mathcal{V}}|} = \frac{1 + |\rho|}{1 - |\rho|} \tag{2.22}$$

where $|\vec{\mathcal{V}}|$ and $|\overleftarrow{\mathcal{V}}|$ are the ordinary amplitudes of the waves travelling in the two directions. When $|\rho|$ is small this relation gives $|\rho| \simeq (S - 1)/2$.

The above treatment of discontinuities by means of circuit theory, represents an oversimplification. We shall pass on to consider further effects which may become of importance at the high frequency end of the nanosecond range.

2.2.4.2 Higher Modes

In addition to the principal mode, corresponding to the simplest field pattern, an infinite variety of field configurations can be found (see BONDI and KUHN [2054]) which satisfy MAXWELL's equations between the conductors and the boundary conditions upon them. The field pattern, in the case of the coaxial line, for the next

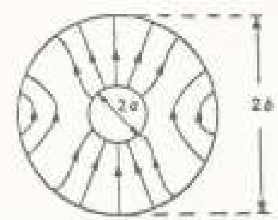


Fig. 2.4. Electric field pattern in a coaxial line for the next mode above the principal.

mode above the principal is shown in fig. 2.4. The general properties of these higher modes are as follows:

(i) The electric and magnetic fields are not perpendicular to the direction of propagation.

(ii) The phase velocity of propagation depends on the frequency.

(iii) There exists a certain cut-off frequency for each mode such that there is no real transmission of energy down the line at lower frequencies; the fields die away and become negligible at a distance down the line of the order of its diameter.

The H_{11} or TE_{11} mode illustrated has the lowest cut-off frequency f_c and the corresponding plane wavelength λ_c is given approximately by

$$\lambda_c = \frac{c}{f_c\sqrt{\kappa_e \kappa_m}} \simeq \pi(a + b)$$

where c is the velocity of light.

Consider the discontinuity indicated in fig. 2.5(a). It is impossible to match the fields on the two sides of the discontinuity simply by combining waves in the principal mode. Higher modes are called into play in order that the boundary conditions and field continuity conditions may be satisfied everywhere. At frequencies below their respective cut-offs the higher modes cannot transmit energy and consequently part of the incident wave is reflected. The effect is more pronounced at the higher frequencies and the high frequency components in the incident signal will be partially reflected with a consequent distortion of the main wave.

Reflection occurs even if the characteristic impedance of the line is the same on both sides of the discontinuity. When it is

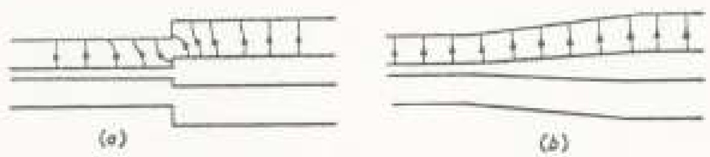


Fig. 2.5. (a) Field pattern in coaxial line involving higher modes at a discontinuity.
(b) Smooth transition from one set of transverse dimensions to another without change in characteristic impedance.

desired to effect a change of dimensions, at constant characteristic impedance, the change should be made smoothly. If the length of the tapered portion (fig. 2.5(b)) is several times the diameter of the outer conductor, and the ratio of the outer to inner diameters is maintained constant, the fields will not sensibly depart from the principal mode type (a pair of coaxial cones can support a principal wave at constant impedance). The reflection coefficient is very small but, as a further refinement, attention may also be paid to the transitions between the coaxial lines and the conical section (see DAHLMAN [2055]).

WHINNERY *et al.* [2056, 2057] and MILES [2058] (also see BROCKELSBY [2059]) have shown how the effect of various types of

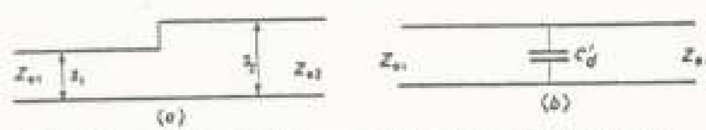


Fig. 2.6. (a) Discontinuity in parallel strip transmission line.
(b) Equivalent circuit.

discontinuity may be represented, for principal mode circuit theory to apply, by a shunt capacitance connected at the junction. Thus, for example, the parallel plate line of fig. 2.6(a) is equivalent to that shown schematically in fig. 2.6(b) where the capacitance C'_d is given by fig. 2.7.

If a uniform dielectric is present these values for the capacitance must be multiplied by κ_e. When dielectric material is to be found

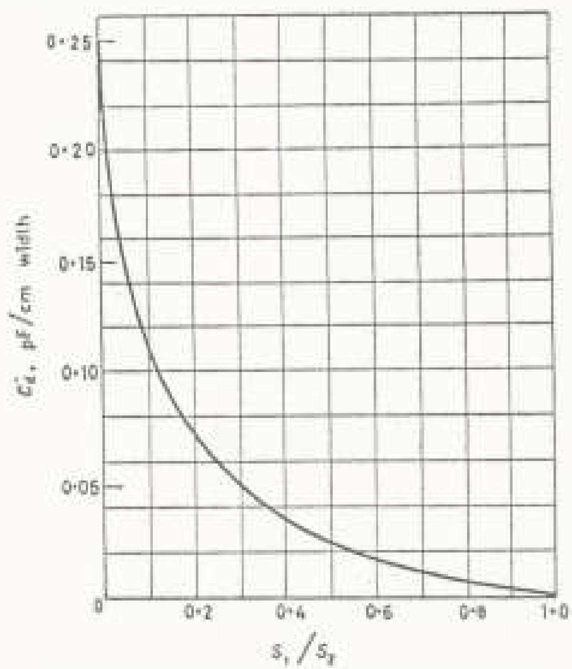


Fig. 2.7. C'_d as a function of S_1/S_2 (see Fig. 2.6).

only on one side of the boundary it is taken into account if it lies on the side where the field disturbance is the greater. Thus, in fig. 2.8(a)

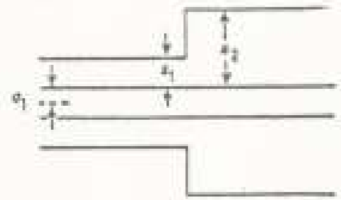
Fig. 2.8. Effect of dielectric on one side of a discontinuity. C'_d is multiplied by κ_e in (a) but not in (b).

the value of C'_d is multiplied by κ_e but is left unaltered in the case of fig. 2.8(b).

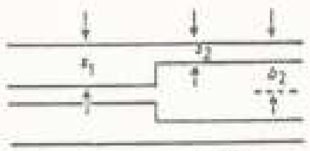
For a coaxial line the equivalent shunt capacitance C_d is given by

$$C_d = 2\pi r C'_d$$

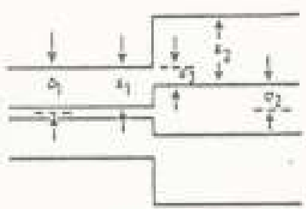
where C'_d is the value tabulated above, per unit width, for parallel plates. The representative radius r for a number of cases is quoted in fig.2.9.



$$C_d \simeq 2\pi a_1 C'_d\left(\frac{s_1}{s_0}\right)$$



$$C_d \simeq 2\pi b_1 C'_d\left(\frac{s_2}{s_1}\right)$$



$$C_d \simeq 2\pi b_1 C'_d\left(\frac{s_2}{s_1}\right) + 2\pi a_2 C'_d\left(\frac{s_3}{s_0}\right)$$

Fig. 2.9. Capacitances equivalent to discontinuities in coaxial line.

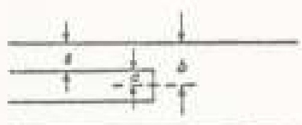


Fig. 2.10. Transmission line with break in inner conductor.

An open circuit termination (fig. 2.10) is equivalent to an ideal line terminated by a capacitance of value given by fig. 2.11 (also see PFIRRMANN [2060]).

Step discontinuities in radial transmission lines are dealt with by BRACEWELL [2061] and by WHINNERY and STINSON [2062].

The equivalent capacitance concept must be applied with the following reservations:

(i) The results are limited to frequencies where the plane wavelength is somewhat greater than the largest transverse

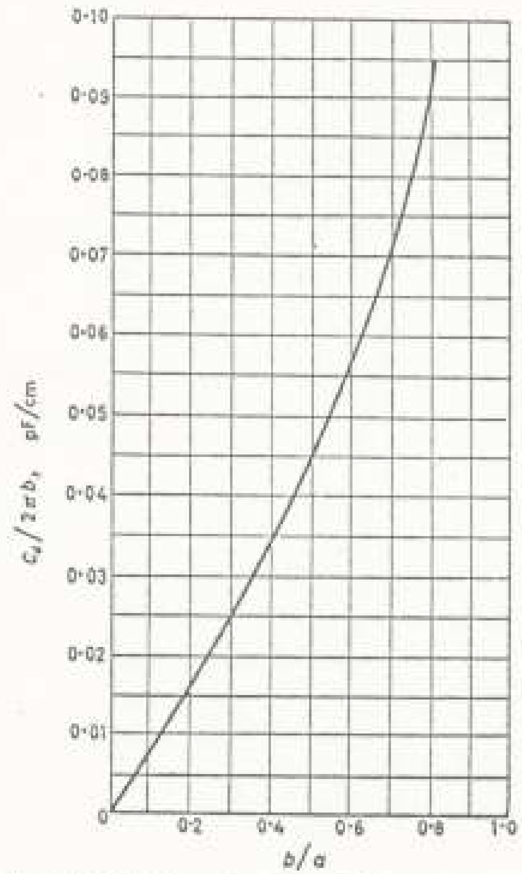


Fig. 2.11. $C_d/2\pi b$ as a function of b/a (see Fig. 2.10).

dimension. When the frequency approaches the cut-off value for one of the higher modes the value of C_d ceases to be frequency independent.

(ii) If multiple discontinuities are present close to one another mutual interaction occurs and the equivalent capacitances are modified (see MARCUS [2063]).

(iii) Similarly the results must be corrected if the line is terminated near a discontinuity ("near" implying within a distance of the order of the greatest transverse dimension).

The distributed circuit methods of § 2.2.4.1 only apply exactly to cases in which the fields at the discontinuity can be accurately matched in terms of waves in the principal mode. The only two cases are:

(i) A change in dielectric material, in what would otherwise be a uniform line, provided the boundary lies in the transverse plane.

(ii) A short-circuit termination, provided this consists of a flat plate or disc placed perpendicularly across the end of the line.

2.2.4.3 Supports

Supports of dielectric material are sometimes employed in the construction of coaxial cable. If these are equally spaced strong reflections can occur at frequencies for which the supports are an integral number of half-wavelengths apart. Such types should be avoided unless the supports are very close together.

Supports are often unavoidable and Cornes [2064] has shown how the reflection coefficient may be made negligible. As illustrated in fig. 2.12(a), the diameter of the inner conductor may be reduced by an amount sufficient to compensate for the influence of the dielectric on the characteristic impedance. This is perfectly satisfactory at medium frequencies but at the higher frequencies the effect of the discontinuity in radius becomes important especially if many supports are employed. If l is the thickness of the support, the equivalent circuit is as shown in fig. 2.12(b) where is Z_0' the characteristic impedance of the short length of line constituted by the support. Clearly l should be as small as possible and the dielectric constant of the insulator should approach unity. Circuit analysis shows that Z_0' should be about 8% greater than Z_0 in a typical case in order to offset the effect of the capacitances. The voltage reflection coefficient is found to be less than 1% for all frequencies up to 4000 Mc/s.

Kaden and Ellenberger [2065] and Peterson [2066] recommend a support with concave faces and describe a modification in which the unwanted shunt capacitance is compensated by the inclusion of a series inductance. This is achieved by undercutting

the inner conductor* in the manner indicated in fig. 2.12(c). The extent of the undercut was determined experimentally and a reflection coefficient of less than 0·4% was obtained up to a frequency of 1200 Mc/s. In a line containing 20 supports, spaced half a wavelength apart, the voltage standing wave ratio was reduced from 2·1 to 1·05 by the introduction of the compensation.

It is evident from the foregoing that the number of plug and socket joints in a run of cable should be kept to a minimum and

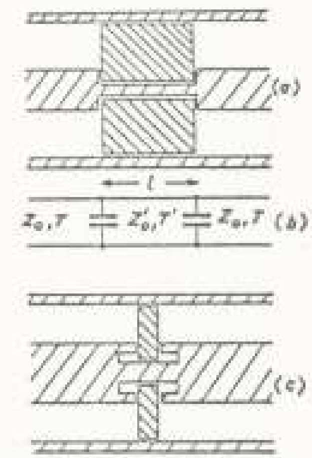


Fig. 2.12. (a) Dielectric support.
(b) Circuit equivalent to (a).
(c) Compensated disc support.

those that are unavoidable should be designed to give a good match over a wide band of frequencies (see, for example, Turner [2067] and Larsen and Haas [2068]). The junction will consist of short lengths of line, of differing impedances, together with the shunt capacitances C_d representing the effect of abrupt changes in dimensions. If the wavelength is much longer than the linear dimensions of the junction (as will be the case in the millimicrosecond region) then each section of length l_n, inductance per unit length L_n, and capacitance per unit length C_n, may be treated as a lumped circuit possessing series inductance l_nL_n and shunt capacitance $l_nC_n + C_{dn}$.

When a short length of line, of characteristic impedance Z_0',

* Such an undercut inner conductor may be useful for other matching purposes and is discussed further by Rowland [2069]. (The slightly higher value of impedance Z_0' used by Cornes gives a compensatory series inductance; see § 2.2.5.3.)

and total delay time T (single transit), is inserted in a line of impedance Z_0, it may be shown that a voltage pulse of amplitude $2T(Z_0' - Z_0)/t_r(Z_0' + Z_0)$ is reflected when the input pulse is of unit amplitude and has a rise-time t_r. This formula, which obtains when $2T < t_r$ and $Z_0' \sim Z_0$, may be applied in the case of a connector. A similar result is quoted by GARWIN [2070].

2.2.4.4 *Terminal Matching*

We have seen that ideally no reflection occurs when a line is terminated by a resistance equal in value to its characteristic impedance. The connexion of a lumped resistor (see MERTEN [2071]) across the line introduces stray reactances which spoil the match at high frequencies; an improvement can be obtained by attempting to satisfy the field boundary conditions by employing a disc type resistor. CROSBY and PENNYPACKER [2072] have studied the behaviour of resistors of the type where the resistive element is in the form of a film deposited on an insulating rod (also see KOHN [2073] and BURKHARDTSMAIER [2074]).* The resistor is housed in a cylindrical container, of length much greater than its diameter, short-circuited at one end. The lossy transmission line so formed (see § 2.2.6) is analysed and the input impedance determined. The total resistance R should be equal to the characteristic impedance of the line to be matched and it is found that optimum performance is obtained when $R/Z_0' = \sqrt{3}$ where Z_0' is the nominal characteristic impedance of the lossy section. A voltage reflection coefficient of about 1% is then found when the ratio of length to wavelength is 0·05; performance improves as this ratio decreases. The length must, however, remain considerably greater than the diameter in order that principal mode theory, which has been used, may apply. KOHN [2075] shows that a much better performance can be obtained if the factor $\sqrt{3}$ above is raised to about $\sqrt{4{\cdot}5}$ provided additional steps are taken to compensate for the reactive component. This may be achieved by an undercut in the line, or by the use of a special compensating line. The frequency range is several times greater than with the uncompensated "$\sqrt{3}$" jacket; a reflection coefficient of $\frac{1}{2}$% is found for a ratio of length to wavelength as high as 0·125.

A design due to HARRIS [2076] employs a uniform cylindrical film resistor housed in a critically dimensioned outer conductor in the

* Further details of resistive components are given in the section on attenuators § 4.6.

shape of a tractrix. An impedance within 1% of the d.c. resistance, and with very small phase angle, was found to obtain from zero frequency up to at least 3450 Mc/s.

In a further paper by KOHN [2077] it is also shown how the boundary conditions at both the surface of the jacket and the resistor may be more accurately met (by a purely TEM wave) by using a tractorial shaped jacket.

Papers by HERSCH [2078] and BAUER [2079] may also be consulted. CLEMENS [2080] treats a similar problem and gives the theory of a lossy exponential line; the arrangements are intended primarily for use in the microwave region, however.

2.2.4.5 *Other Types of Discontinuity*

Further miscellaneous information on the effects of irregularities and discontinuities is given in papers by BLACKBAND [2081], COX [2082], FUCHS [2083], RAYMOND [2084] and SAFIR [2085] (also see § 8.3).

KING and TOMIYASU [2086] give an account of the effect of the geometry of the connexion to the load. In practice, terminal discontinuities are equivalent to a shunt capacitance connected across the line together with an inductance in series with the idealized lumped load.

TOMIYASU [2087] finds that the effect of a sharp bend in a transmission line is equivalent to the insertion of a shunt capacitance, followed by a series inductance, which in turn is succeeded by a second shunt capacitance. OLIVER [2088] gives an account of the influence of discontinuities in slotted concentric line impedance measuring gear over the range 100–3000 Mc/s.

MARCHAND [2089, 2090] enumerates a number of theorems concerning breaks in the outer shield of a screened line. At frequencies greater than 50 Mc/s the shield can be assumed to be perfect in the sense that the current on the inside surface and that flowing on the outside surface (if any) are quite independent of each other. KIRCHHOFF's laws may then be applied at the break in order to relate the currents.

2.2.5 Transmission line as a Circuit Element.

Relations 2.1, 2.6, 2.7, 2.15 and 2.17 together with the discontinuity equivalents given above tell us all we need to know concerning the properties of a transmission line as a circuit element. In the applications to be described we shall see how the three basic properties which a line possesses namely (i) time delay, (ii) resistive characteristic impedance and (iii) reflection phenomena, are turned to advantage. The extensive use of lines in nanosecond equipment is due to the fact that the lengths involved are not inconveniently great as would usually be the case in the microsecond region (also see § 2.5).

2.2.5.1 Expression for Voltage at Input End

Fig. 2.13 depicts a source of e.m.f. $\bar{E}$, of internal impedance Z_1, situated at $x = 0$, feeding a length l of line, of characteristic im-

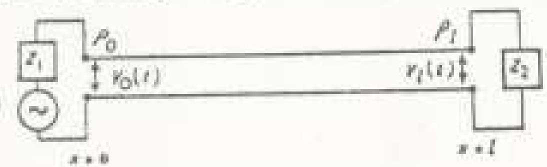


Fig. 2.13. General case for analysis.

pedance Z_0, which is terminated at the point $x = l$ by an arbitrary impedance Z_2. We require to find the voltage on the line at the input end.

From equation 2.9 the total voltage $\bar{V}_0$ at $x = 0$ consists of the sum of two wave components $\vec{V}_0 + \overleftarrow{V}_0$. The former is made up of (i) a contribution due to the applied e.m.f. of amount $\bar{E} \cdot Z_0/(Z_1 + Z_0)$ given by the elementary potential divider formula and (ii) the fraction $\rho_0 \cdot \overleftarrow{V}_0$ of the wave incident from the right which is reflected at $x = 0$. Here ρ_0 is the voltage reflection coefficient at the near end for waves incident from the right. Thus

$$\vec{V}_0 = a \cdot \bar{E} + \rho_0 \cdot \overleftarrow{V}_0 \qquad (2.23)$$

where we have put

$$a = \frac{Z_0}{Z_1 + Z_0} \qquad (2.24)$$

the straightforward attenuation factor.

From relations 2.6, 2.7 and 2.17 we have

$$\frac{\overleftarrow{V}_0}{\vec{V}_0} = \rho_l e^{-2pTl} \qquad (2.25)$$

On solving for $\vec{V}_0$ and $\overleftarrow{V}_0$ from relations 2.23 and 2.25 we obtain

$$\vec{V}_0 = a \cdot \frac{1}{1 - \rho_0\rho_l e^{-2pTl}} \cdot \bar{E} \qquad (2.26)$$

and

$$\overleftarrow{V}_0 = a \cdot \frac{\rho_l e^{-2pTl}}{1 - \rho_0\rho_l e^{-2pTl}} \cdot \bar{E} \qquad (2.27)$$

hence

$$\bar{V}_0 = \vec{V}_0 + \overleftarrow{V}_0 = a \cdot \frac{1 + \rho_l e^{-2pTl}}{1 - \rho_0\rho_l e^{-2pTl}} \cdot \bar{E} \qquad (2.28)$$

This general result can be interpreted in familiar terms if the denominator is expanded* as a power series; thus

$$\bar{V}_0 = a(1 + \rho_l e^{-2pTl})[1 + \rho_0\rho_l e^{-2pTl} + (\rho_0\rho_l)^2 e^{-4pTl} + \ldots] \cdot \bar{E} \qquad (2.29)$$

When the inverse transform is taken, $V_0(t)$ is seen to consist of the voltage $a \cdot E(t)$ up to time $t = 2Tl$ followed by an added contribution $a\rho_l(1 + \rho_0) \cdot E(t - 2Tl)$ for $2Tl \leq t < 4Tl$. At time $t = 4Tl$ a further contribution of $a\rho_0\rho_l^2(1 + \rho_0) \cdot E(t - 4Tl)$ is added and so on. The extra contribution which appears when $2nTl \leq t \leq 2(n + 1)Tl$ is equal to $a(\rho_0\rho_l)^{n-1}\rho_l(1 + \rho_0) \cdot E(t - 2nTl)$ where n is an integer.

Relations 2.28 and 2.29 are quite general but in the interpretation just given we have supposed that both ρ_0 and ρ_l are independent of p (i.e. both Z_1 and Z_2 are pure resistances) when taking the inverse transform.

2.2.5.2 Pulse Shaping

Let us restrict ourselves to the case in which the line is matched at the source end. Then $Z_1 = Z_0$, $\rho_0 = 0$ and $a = 1/2$. Equation 2.28 then becomes

$$\bar{V}_0 = \tfrac{1}{2}(1 + \rho_l e^{-2pTl}) \cdot \bar{E} \qquad (2.30)$$

No multiple reflection takes place and there is only one extra contribution to $V_0(t)$ which occurs at and after time $t = 2Tl$. Three particular cases, already mentioned in § 2.2.4.1, are of importance:

(a) $Z_2 = Z_0$. Here $\rho_l = 0$ and $V_0(t) = E(t)/2$

(b) $Z_2 = 0$. In this case $\rho_l = -1$ and

$$V_0(t) = \tfrac{1}{2}[E(t) - E(t - 2Tl)]$$

* This expansion is rigorously justified since p is a positive quantity and the reflection coefficients are each less than or equal to unity.

This arrangement forms a very useful clipping circuit, which is superior to the differentiating circuit of fig. 1.5(b), since the portion of the pulse between $t = 0$ and $t = 2Tl$ is reproduced without any modification in shape.

(c) $Z_2 \to \infty$. Now $\rho_l = 1$ and accordingly relation 2.30 yields

$$V_0(t) = \tfrac{1}{2}[E(t) + E(t - 2Tl)]$$

The results for a unit step function input and for a ramp function input of rate of rise m are shown in fig. 2.12(a), (b). The waveforms

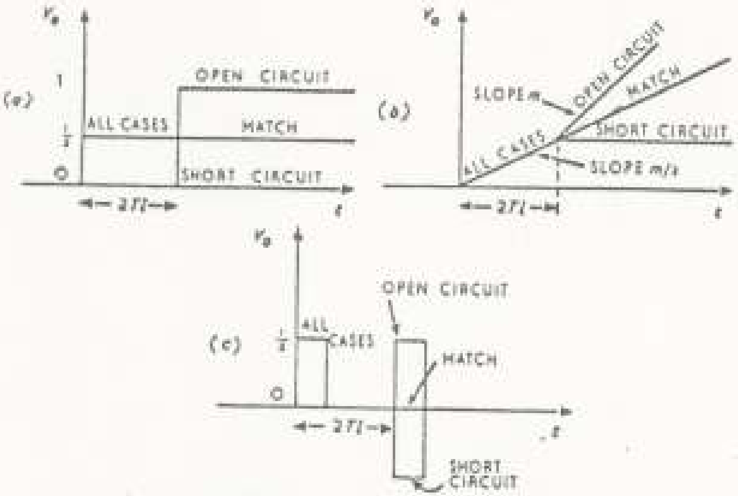


Fig. 2.14. Voltage at input of line with various types of termination.
(a) unit step function input
(b) ramp function input, slope m
(c) short rectangular pulse input
Source matched in all cases.

obtained with a short rectangular pulse of duration less than $2Tl$ (which are, of course, derived from the results for a step function) are depicted in fig. 2.14(c). It is seen that two pulses have been obtained from one with a relative delay between them of twice the transit time down the line. Such an artifice may be applied, for example, in a single channel double pulse generator unit.

When either the source or the terminating impedance is partially or wholly reactive the reflection coefficients involve p and this must

be borne in mind when taking the inverse transform of equation 2.28. The results for a number of cases are summarized in fig. 2.15.

2.2.5.3 *Input Impedance*

The expression for the input impedance $Z_{\rm in}$ of a finite length of line, arbitrarily terminated, follows from the results of the preceding section. By the elementary potential divider formula

$$V_0 = \frac{Z_{\rm in}}{Z_{\rm in} + Z_1} \cdot E \qquad (2.31)$$

which, on using relations 2.28, 2.24 and 2.17, yields

$$Z_{\rm in} = Z_0 \frac{1 + \rho_l e^{-2pTl}}{1 - \rho_l e^{-2pTl}} \qquad (2.32)$$

This result has turned out to be independent of Z_1 as should indeed be the case if the input impedance is to have a definite significance. It is noted immediately that, if the line is correctly terminated at the far end ($\rho_l = 0$), then $Z_{\rm in} = Z_0$ as expected.

We shall restrict ourselves to one special case, out of many of interest, and find the lumped circuit to which an arbitrarily terminated short length of line is equivalent. If $\omega Tl \ll 1$ we can write $e^{-2pTl} \simeq (1 - 2pTl)$ and equation 2.32 reduces to

$$Z_{\rm in} \simeq Z_2 \frac{1 - pTl(Z_2 - Z_0)/Z_2}{1 + pTl(Z_2 - Z_0)/Z_0} \qquad (2.33)$$

where we have substituted for ρ_l in terms of Z_2 and Z_0.

(a) If $Z_0 = Z_2$ then $Z_{\rm in} = Z_2$.

(b) When $Z_0 \gg Z_2$ relation 2.33 gives

$$Z_{\rm in} \simeq Z_2(1 + pTlZ_0/Z_2) = Z_2 + pLl$$

on using equations 2.8 and 2.14. The effect of the line is to add a series lumped inductance equal to its total natural inductance.

(c) In the case $Z_0 \ll Z_2$ we find

$$Z_{\rm in} \simeq \frac{Z_2}{1 + pTl/Z_0}$$

which is the expression for the combination of the impedance Z_2 in parallel with the total natural capacitance Cl of the line.

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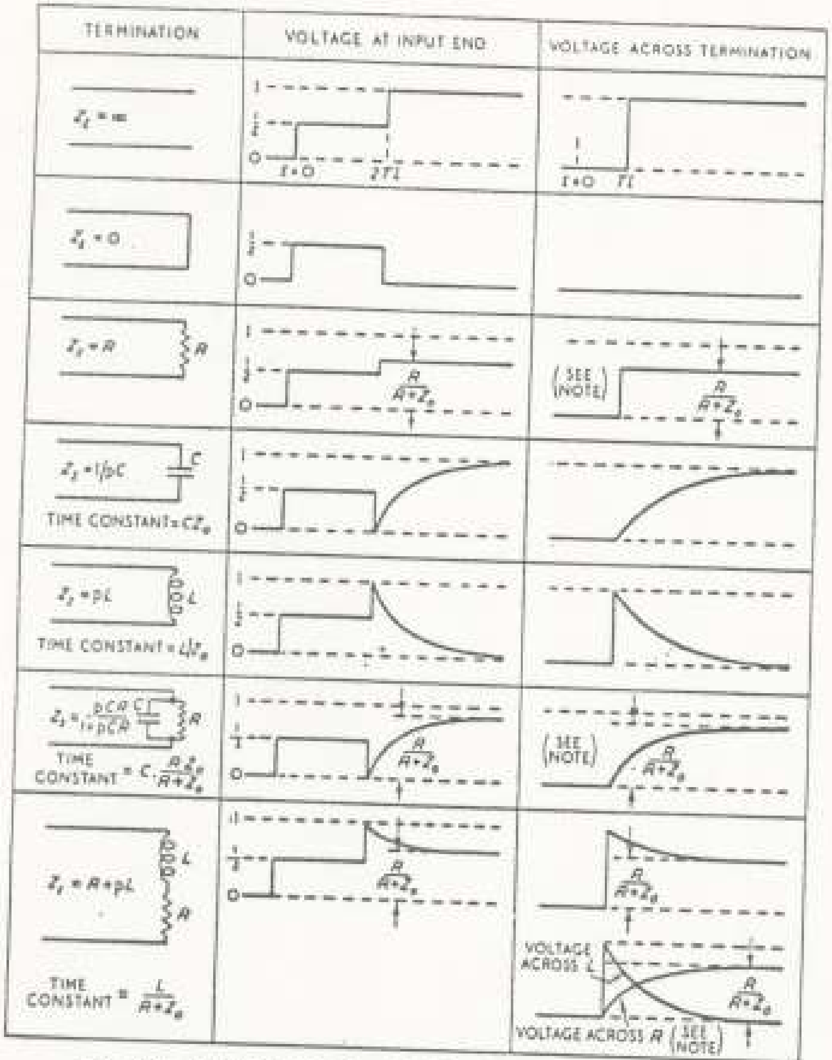

Fig. 2.15. Circuit properties of a length l of lossless uniform transmission line of characteristic impedance Z_0 when fed from a matched source giving a unit step function of e.m.f. at time $t = 0$.

NOTE: In these cases when R represents the input impedance of a second length of transmission line the signal launched in this line is of identical form.

We are also interested in the general expression for the voltage $\bar{V}_l$ which appears across the termination.

Now

$$\bar{V}_l = \vec{V}_l + \overleftarrow{V}_l = (1 + \rho_l)e^{-pTl} \cdot \vec{V}_0 \tag{2.34}$$

On applying relation 2.26 this becomes

$$\bar{V}_l = a \frac{(1 + \rho_l)e^{-pTl}}{1 - \rho_0\rho_l e^{-2pTl}} \cdot \bar{E} \tag{2.35}$$

which, by virtue of equation 2.28, may be written in terms of the resultant voltage at the input

$$\bar{V}_l = \frac{(1 + \rho_l)e^{-pTl}}{1 + \rho_l e^{-2pTl}} \cdot \bar{V}_0 \tag{2.36}$$

2.2.6 Losses.

Throughout the previous sections we have assumed that the transmission line was completely lossless. Losses may arise in practice due to:

(a) The finite resistance of the conductors.

(b) The conductivity of the dielectric (usually utterly negligible) and the radio-frequency losses therein.

(c) Radiation in the case of open, unscreened, lines.

It is usually assumed that if the losses are small the field patterns do not depart appreciably from those appertaining to the principal mode, and it turns out that all losses are equivalent to a distributed series resistance R and shunt conductance G per unit length (see Buchholtz [2013] for example).

2.2.6.1 Effect on Propagation and Impedance

The basic equations 2.2 and 2.3 become

$$\frac{d\bar{V}_x}{dx} = -(R + pL)\bar{I}_x \tag{2.37}$$

$$\frac{d\bar{I}_x}{dx} = -(G + pC)\bar{V}_x \tag{2.38}$$

We note that all our previous relations involving the Laplace transforms of the variables still stand provided L is replaced by $L(1 + R/pL)$ and C by $C(1 + G/pC)$ throughout. If the frequency is high enough, as will normally be the case in the millimicrosecond range, such that $R \ll \omega L$ and $G \ll \omega C$ relations 2.8 and 2.14 become

$$T \simeq \sqrt{LC}\left[1 + \left(\frac{R}{L} + \frac{G}{C}\right)\Big/2p\right] \tag{2.39}$$

$$Z_0 \simeq \sqrt{\frac{L}{C}}\left[1 + \left(\frac{R}{L} - \frac{G}{C}\right)\Big/2p\right] \tag{2.40}$$

The solution 2.6 for a wave travelling to the right now reads

$$\vec{V}_x = \vec{V}_0\varepsilon^{-pTx} = \vec{V}_0\varepsilon^{-\alpha x}\varepsilon^{-p\sqrt{LC}x} \tag{2.41}$$

where

$$\alpha = \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}} \tag{2.42}$$

We see that the delay time is unaffected, to the first order of terms in $1/p$, and an attenuating factor $\varepsilon^{-\alpha x}$ has been introduced. The characteristic impedance has a value which differs from the old by a term in $1/p$.

2.2.6.2 Sources of Loss

When MAXWELL's equations are solved for a wave propagated in and over the surface of a metal of finite conductivity it is found that the field intensity falls off exponentially with distance below the surface. The current density is reduced to a fraction $1/\varepsilon$ of its surface value at the skin depth d metres given by

$$d = \frac{1}{\sqrt{\pi f \mu \sigma}} \tag{2.43}$$

where f is the frequency* in c/s, σ is the conductivity in mhos/m and $\mu = \kappa_m \mu_0$ is the permeability of the conductor. At a frequency of 1 Mc/s, for example, with a copper conductor, $d \simeq 6 \times 10^{-3}$ cm and thus, in the millimicrosecond range, the currents may be assumed to flow entirely over the surface of the conductors. For the coaxial line, with copper conductors radii a and b, we have

$$R = 4{\cdot}2 \times 10^{-8}\sqrt{f}\left(\frac{1}{a} + \frac{1}{b}\right)\ \Omega/\text{m} \tag{2.44}$$

* Treatments of the skin effect for pulse signals are given by MILLER [2091], SIMM [2092, 2093] and VALLESE [2094, 2095].

The effective resistance increases with frequency but the ratio $R/\omega L$ continues to decrease as the frequency is raised thus justifying the approximations 2.39 and 2.40. The attenuation is seen to rise with frequency resulting in a loss of the higher signal component frequencies. The distortion only appears as a slowing in the rise-time, however, since there is no accompanying phase distortion (to the first order). A comprehensive collection of formulae and graphs for the skin effect in conductors of various shapes composed of different materials is given by WHINNERY [2096]; also see WHEELER [2097, 2098] and VARCHENYA and TYUTIN [2099].

It may be noted that skin resistance may be reduced by plating the conductors with silver to a thickness equal to several times the skin depth. It may also be mentioned that the resistance of an outer conductor (shield) composed of thin wires braided together may be several times that of the corresponding metal tube and the variation with frequency seems to be greater, at high frequencies, than predicted by simple skin effect theory.

It is possible to choose the line dimensions such that the attenuation is a minimum. For a coaxial line the attenuation constant has a flat minimum when $b/a = 3{\cdot}6$ i.e. when $Z_0 = 77\ \Omega$. This and other optimum arrangements are discussed by SMITH [2100] and BLACKBAND [2101]. It is suggested that the best choice of impedance is 75 Ω for low-loss air-spaced cables, and 50 Ω for general purpose thermoplastic cables. Such considerations are irrelevant however except when long distances or high powers are involved.

It might well be supposed that the fact that R and G vary with frequency would make it impossible to obtain a rigorous circuit solution, using Laplace transform methods, for the transient case. However, a sinusoidal analysis, in terms of electromagnetic fields, reveals that the effective series inductance per unit length is increased by an amount numerically equal to R/ω where R, given by relation 2.44, is of the form $k\sqrt{f}$ (ω is the angular frequency). In other words the series impedance per unit length becomes

$$k\sqrt{f}(1 + j) + j\omega L$$

Invoking the correspondence between sinusoidal and Laplace transform analysis whereby $p = j\omega$, and noting that $\sqrt{j} = (1 + j)/\sqrt{2}$, we find that the series impedance may be written as $2\sqrt{\pi}k\sqrt{p} + pL$. WIGINGTON and NAHMAN [2102] use this expression in the transmission line equations (2.37) and (2.38) and readily obtain a formula for the propagation constant. By expanding and neglecting powers in $1/p$ the inverse transform is readily obtained. It is found that a unit step function input pulse, after traversing a length l of cable, has a shape given by:

$$1 - \mathrm{erf}\left(\frac{l\alpha_0}{2\sqrt{\pi f_0 t}}\right) \qquad (2.45)$$

where α_0 is the attenuation (nepers per metre) at some particular frequency f_0 (c/s), and t is the time (sec) excluding the normal delay

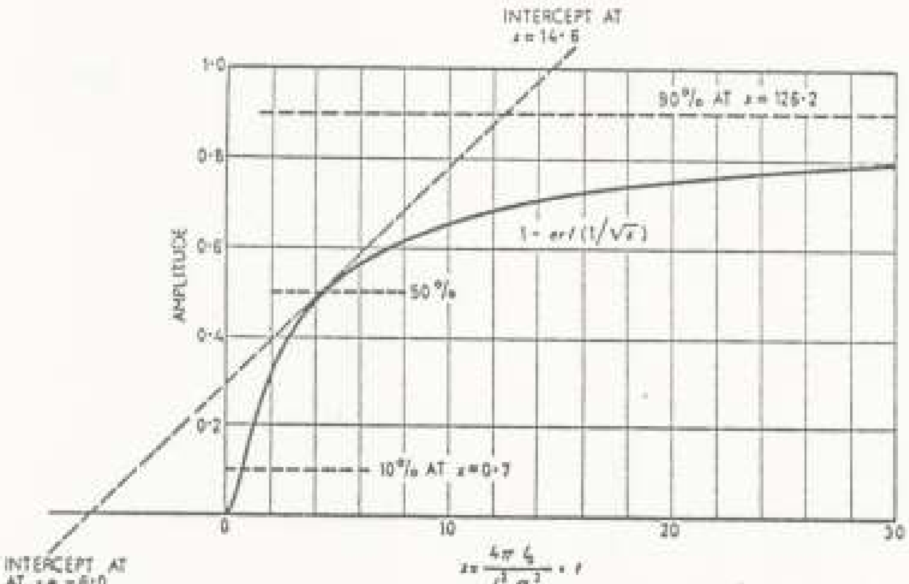

Fig. 2.16. Calculated distortion suffered by a unit step function on traversing a length of coaxial cable.

down the cable. The majority of cables follow the law $R = k\sqrt{f}$ quite closely but it is advisable to choose the particular frequency f_0, at which α_0 is determined, near the top end of the frequency band of interest. The function (2.45) is plotted in fig. 2.16 with a normalized time-scale.

It is interesting to compare the rise-time taken from fig. 2.16 with that predicted by relation (1.5) given in Chapter 1. If α_c is

the attenuation constant at the "cut-off" frequency i.e. at which the amplitude falls to $1/\sqrt{2}$ of its normal value, then

$$e^{-\alpha_c l} = 1/\sqrt{2}$$

Now

$$\alpha_c = \alpha_0\sqrt{(f_c/f_0)}$$

thus we find:

$$f_c = \frac{f_0}{\alpha_0^{2}l^{2}}(\ln 0{\cdot}707)^2$$

We then apply the relation $t_r = 0{\cdot}45/f_c$ with the result tabulated below. Owing to the very slowly rising top, it is not easy to assess the rise-time of the waveform of fig. 2.16 very precisely. The rise-time as defined between the 10% and 90% points seems on the pessimistic side, whilst the tangent-intercept definition gives an over optimistic estimate of performance. The results based on the two definitions are read off from fig. 2.16 and included in Table 2.1.

TABLE 2.1. Pulse distortion due to loss in coaxial cable.

Measurement	*Rise-time*
$0{\cdot}45/f_c$	$3{\cdot}74 \times \alpha_0^{2}l^{2}/f_0$
10–90%	$9{\cdot}99 \times \alpha_0^{2}l^{2}/f_0$
tangent-intercept	$1{\cdot}64 \times \alpha_0^{2}l^{2}/f_0$

It is seen that the value given by equation (1.5) falls between the two extremes and is thus adequately justified as a rough practical measure.

A contribution by TURIN [2103] supplements the work by WIGINGTON and NAHMAN, and KERNS [2104] reports measurements on a number of different types of cable which support the theory outlined above. The assumptions are valid between (i) a low frequency at which the skin depth is equal to the thickness of one or both conductors, and (ii) an upper frequency at which dielectric losses become comparable with skin effect loss. The latter occurs at about 1000 Mc/s for solid polythene dielectric cables, at a value greater than 1500 Mc/s for RG63, and at over 4000 Mc/s for Styrofoam-filled cable.

Further information on transient distortion in a uniform line is given by CAZENAVE [2105].

The principal contribution to the *shunt conductance* G is that of the ordinary dielectric loss in the material given by

$$G = \omega C \tan \delta \qquad (2.46)$$

where $\tan \delta$ is approximately equal to the power factor when the losses are small. A list of permittivities and loss factors for various materials is quoted by MORENO [2106], for example, and the relative merits of certain substances are discussed by WILLIAMS and SCHATZ [2107]; BRECKINRIDGE and THURNAUER [2108] have compiled a digest of the literature on dielectrics.

TABLE 2.2. Data for typical coaxial cables. Length for 100 nsec delay ~ 80 ft.

Type		Characteristic impedance Z_0 ohms	Overall diameter inches	Attenuation in db for 100 nsec delay				
British	American approximate equivalent			10 Mc/s	30 Mc/s	100 Mc/s	300 Mc/s	1000 Mc/s
Uniradio 39	RG/39U	69	0·31	0·47	0·86	1·65	3·2	7·6
Uniradio 60	RG/13U	75	0·46	0·36	0·64	1·2	2·1	4·5
T.C.M.–AS50	RG/62U	100	0·28	0·58	1·0	1·9	3·3	7·0
Transradio —C3-T	—	197	0·64	0·43	0·85	1·6	3·6	10·2
Uniradio 85	—	75	0·47	0·23	0·39	0·72	1·28	2·5

The attenuation found in practice, taking all sources of loss into account, is indicated in Table 2.2 for a number of different varieties of coaxial cable.

The Uniradio types 39 and 60 have solid polythene insulation and the T.C.M. type AS50 and Transradio C3-T are semi air-spaced (see GRIEMSMANN [2109]). All these cables are mechanically flexible and long lengths can be stored conveniently. Uniradio 85, on the other hand, has a solid aluminium outer conductor with helical polythene spacer—hence the low attenuation found.

Painting the conductors with a protective coating, such as lacquer, tends to introduce dielectric loss but the protection of the metal surface from oxidation and corrosion, which increase the skin resistance, more than outweighs the disadvantage.

2.2.6.3 *Laminated Construction*

A new departure in transmission line construction, designed to give small attenuation, is suggested by CLOGSTON [2110]. The inner conductor of a coaxial line, for example, consists of a central insulating core surrounded by numerous thin cylindrical layers of metal and dielectric disposed alternately. The whole is enclosed in the usual cylindrical outer shield with the usual insulating material separating the inner stack from the shield. The stack has a certain average permittivity for transverse electric fields and it is shown that the wave, and accompanying currents, penetrate most deeply into the stack (this being required to reduce the effective resistance) if the wave velocity on the line is equal to that associated with this average permittivity. This may be brought about by a suitable choice of the main bulk of insulating material.

A transmission line completely filled with laminae is discussed and an analysis is given of the modes of transmission of such a system and of the problem of terminating the line. The theory of planar arrangements is developed for both infinitely thin laminae and for laminae of finite thickness. Experimental work on a partly laminated line is reported by BLACK *et al.* [2111].

Further information on lines of this type is contained in papers by KADEN and MARTIN [2112], KING and MORGAN [2113], MARTIN [2114], MORGAN [2115], VAAGE [2116] and WILD *et al.* [2117].

ATAKA [2118] suggests the use of a conductor which is laminated and insulated at *right angles* to the normal direction of current flow.

2.3 HELICAL LINES

2.3.1 Introduction.

Rectilinear transmission lines, operating in the principal mode, have the outstanding advantage over other types that the frequency characteristic is limited only by skin effect and dielectric losses. The major drawback with such lines is the very limited range of characteristic impedances which are available if the transverse dimensions are not to be impracticably large (see graph Appendix III). Also, when long delay times are required, the corresponding lengths of cable may be inconveniently great in some applications. Transmission cables have accordingly been developed in which the inner conductor is in the form of a closely wound, single layer, spiral; this helix is enclosed within the normal cylindrical sheath. In such